CH-2

Linear Algebra.

- In General there are 2 - types of Linear equation.

Linear equation :-

$Ax = B$ [Non- Homogenous]

$Ax = 0$ [Homogenous]

Here, A = Coefficient matrix.

x = unknown matrix (variable)

B = known matrix

Matrix :-

- matrix is the collection of Number (Real or complex).

in fixed no. of Rows/ coloumns.

$[A]_{R\times C} = \{a_{ij}\}_{R\times C}$

Name of matrix

element

No. of Rows. (No. of equation.

Position of element (Rows)

Position of element (column)

No. of Columns. (No. of variables)

$(R \times C)$:- No. of elements = Area of matrix

(Altogether diamensions).

ex :-

$$[A]_{3\times3} = \begin{array}{c} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{array}{l} \to R_1 \\ \to R_2 \\ \to R_3 \end{array} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ C_1 & C_2 & C_3 \end{array} \end{array} \quad 3\times3 \to \text{order of } A$$

Types of matrix A/c to it's dimensions (R, C)

Definition is sufficient condition.
Definition & Prop ↓ Property

Case-(i) $R = C$ (No. of eq^n = no. of variable)

Row = column.

$$[A]_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

$a_{13} \rightarrow i<j$

$a_{31} \rightarrow i>j$

→ Square matrix. (R = C)

Secondary/OFF diagonal

$i=j$ → main/ON/Leading/Primary Diagonal

- If no. of eq^n = no. of variable is called Balanced System/Determined system.

NOTE :- A square matrix is very usefull in Linear Algebra. because diagonal, Determinant, Adjoint, Inverse, unique solution, characterstic eq^n. are only valid for square matrix.

- Definition is sufficient condition and property is neccessary cond^n.

case-(ii) $R \neq C$:- Non square martrix (Rectangular matrix).

unblanced system

(A) $R < C$ (equation < variable)

- under determined system

$$A_{2\times3} = \begin{bmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{bmatrix}_{2\times3}$$

↳ Horizontal matrix

(B) $R > C$ (equation > variable)

- over determined system

$$A_{3\times2} = \begin{bmatrix} 1 & 7 \\ 2 & 8 \\ 4 & 6 \end{bmatrix}_{3\times2}$$

R×C

↳ Vertical matrix

(C) $R = 1$, $C > 1$

$A_{1\times3} = [a_{11} \ a_{12} \ a_{13}]$ → Row Vector.

(D) $R > 1$, $C = 1$

$$A_{3\times1} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$$ → Colomn vector.

Formation of matrix A/c to position of elements (i,j)

$$A_{2\times 3} = \{a_{ij}\}_{2\times 3} = \begin{cases} i+j & i>j \\ 0 & i=j \\ i-j, & i<j \end{cases}$$

$$A_{2\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \rightarrow i<j$$

$a_{21} \rightarrow i>j$ $\quad$ $a_{11}, a_{22} \rightarrow i=j$

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 3 & 0 & -1 \end{bmatrix}_{2\times 3}$$

GATE

$$A_{n\times n} = \{a_{ij}\}_{n\times n} = \{i^2 - j^2 \;\forall\; i,j\} \text{ then } \sum a_{ij} \text{ is } \underline{0}$$

$\forall$ → for every

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{bmatrix}$$

a_{12}, a_{13}, a_{23} → -ve

a_{21}, a_{31}, a_{32} → +ve

Question for $A_{n\times n} = \{a_{ij}\}_{n\times n}$ find sum of all elements for

(1) $A = \{a_{ij}\} = \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$

* (2) $A = \{a_{ij}\} = \begin{cases} i & i=j \\ 0 & i\neq j \end{cases}$

(3) $A = \{a_{ij}\} = \{1 \;\forall\; i,j\}$

* (4) $A = \{a_{ij}\} = \begin{cases} i+j & i=j \\ 0 & i\neq j \end{cases}$

* (5) $A = \{a_{ij}\} = \begin{cases} i\times j & i=j \\ 0 & i\neq j \end{cases}$

* (6) $A = \{a_{ij}\} = \begin{cases} i^3 & i=j \\ 0 & i\neq j \end{cases}$

(7) $A = \{ i + j \ \forall \ i, j$

(8)* $A = \{a_{ij}\} = \{ i^n - j^n \ \forall \ i, j \}$

$i < j$

$$A_{n\times n} = \{a_{ij}\} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

$i > j$ $\qquad$ $i = j$

Sol^n (1)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{n\times n} \qquad \sum a_{ij} = 1 + 1 + 1 + \ldots + n \text{ times} = n$$

Sol^n (2)

$$A = \begin{bmatrix} i=1 & 0 & 0 \\ 0 & i=2 & 0 \\ 0 & 0 & i=3 \end{bmatrix}_{n\times n} \qquad \sum a_{ij} = 1 + 2 + 3 + \ldots + n \text{ times}$$

$$= \frac{n(n+1)}{2}$$

$$\boxed{\text{A.P,} \quad S_n = \frac{n}{2}(a + l)}$$

Sol^n (3)

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{n\times n} \begin{matrix} \to \sum R_1 \\ \to \sum R_2 \\ \to \sum R_n \end{matrix} \Bigg\} = \sum R_1 + \sum R_2 + \sum R_n$$

$$\sum R_1 = \sum R_2 = \sum R_n = n$$

Total Sum $= n + n + n + \ldots n \cdot \text{times} = n \times n = n^2$

Sol^n (4)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \\ & & & \ddots \\ & & & & 2n \end{bmatrix}_{n \times n}$$

$$\sum a_{ij} = 2(1+2+3+\dots+n) = 2 \times \frac{n}{2}(1+n) = n(n+1)$$

Sol^n (5)

$$A = \begin{bmatrix} 1^2 & & \\ & 2^2 & \\ & & \ddots \\ & & & n^2 \end{bmatrix} \quad \sum a_{ij} = 1^2 + 2^2 + \dots n^2$$
$$= \frac{n(n+1)(2n+1)}{6}$$

Sol^n (6)

$$A = \begin{bmatrix} 1^3 & & \\ & 2^3 & \\ & & \ddots \\ & & & n^3 \end{bmatrix} \quad \sum a_{ij} = 1^3 + 2^3 + \dots + n^3$$
$$= \left[\frac{n(n+1)}{2}\right]^2$$

Sol^n (7)

$$A = \begin{bmatrix} 2 & 3 & 4 & \dots & (1+n) \\ 3 & 4 & 5 & \dots & (2+n) \\ \vdots \\ (n+1) & (n+2) & (n+3) & \dots & (n+n) \end{bmatrix}_{n \times n} \begin{array}{l} \to \sum R_1 = (n+3)\frac{n}{2} \\ \to \sum R_2 = \frac{n}{2}(5+n) \\ \to \sum R_n = \frac{n}{2}[(2n+1)+n] \end{array}$$

$$\sum R_1 = 2 + 3 + 4 + \dots + (1+n) = \frac{n}{2}(3+n)$$

A.P Series.

$$\text{Total sum} = \sum a_{ij} = \frac{n}{2}\left[(3+n) + (5+n) + (7+n) + [(2n+1)+n]\right]$$

$$= \frac{n}{2} * \left[\frac{n}{2}\{(3+n) + 2n+1+n\}\right]$$

$$= \frac{n^2}{4}\{(4+4n)\} = n^2(1+n) \text{ Ans.}$$

Elementry Transformation :-

- To make the element zero we use elementry Xformation.

3 types of elementry Transformation :-

(1) we can Interchange 2 rows or colomns.

ex :- $R_1 \longleftrightarrow R_3$

(2) we can multiply any non-zero no. to Any Line.

ex :- $C_2 \longrightarrow \frac{1}{2} C_2$

(3) we can multiply any non-zero no. to Any Line, and Add or substract with the another parallel Line.

ex :- $R_3 \longrightarrow R_3 - 2R_1$

$\boxed{\ } R_2 \longrightarrow 2R_1 + \boxed{\ } R_2$

same. (det. not changed.)

Q.

$$\{a_{ij}\} = A = \begin{bmatrix} 2 & -3 & 4 \\ 5 & -8 & 1 \\ 7 & 9 & -1 \end{bmatrix}_{3\times 3}$$

follow the E.Ts and find

(i) $R_1 \to R_1 + R_2$

(ii) $R_3 \to R_3 + 3R_1$

(iii) and find $\sum a_{22} + a_{11}$ is ...

$$A = \begin{bmatrix} 7 & -11 & 5 \\ 5 & -8 & 1 \\ 13 & 0 & 11 \end{bmatrix}$$

$$\sum a_{22} + a_{11} = -8 + 7 = -1$$

Basic Operation of Matrix

(1.) Transpose :-

- It is valid for both square and non-square matrix.
- we can get the transpose by Interchanging rows and coloumn.
- It is represented by A^T or A'.

ex :-

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2} \qquad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$$

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 1 \\ 4 & 2 & 4 \end{bmatrix}_{3\times 3} \qquad A^T = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 2 \\ -2 & 1 & 4 \end{bmatrix}_{3\times 3}$$

Interchanging $\boxed{R \longleftrightarrow C}$

$$\{a_{ij}\}_{R\times C} \xrightarrow{T} \{a_{ji}\}_{C * R}$$

- In diagonal element (leading) no change in Transpose of Square matrix.

** - Sum of elements of leading diagonal is known as Trace. (tr).

$$* \boxed{tr(A) = tr(A^T)}$$

- Trace (tr) $= \sum a_{ij}$, $i = j$

(2.) Matrix multiplication :-

- It is valid for both square as well as non-square matrix.

$$[A]_{x * y} \; [B]_{y \times z} = [A\,B]_{x \times z}$$

Row of 1st

Same

Col. of 2nd

* Inner order of Product

outer order of Product

Example :-

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$$

$$C = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2}$$

$$[A\,B]_{2\times 3} = \begin{bmatrix} 2\times1+4\times1 & 2\times2+1\times5 & 2\times3+1\times6 \\ 3\times1+2\times4 & 3\times2+2\times5 & 3\times3+2\times6 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 6 & 9 & 12 \\ 11 & 16 & 21 \end{bmatrix}_{2\times 3}$$

m = 12 Add = 6

$$[B\,C]_{2\times 2} = \begin{bmatrix} 1\times1+2\times2+3\times3 & 1\times4+2\times5+3\times6 \\ 4\times1+5\times2+6\times3 & 4\times4+5\times5+6\times6 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}_{2\times 2}$$

m = 12 Add = 8

Key points :-

(i) For $A_{x\times y} \cdot B_{y\times z} = [A\,B]_{x\times z}$

- $x \times z$ is reffered as outer order and y is reffered as inner order.
- No. of elements (e) = outer order ($x * z$)

$$e = x \times z$$